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1995 J. Phys.: Condens. Matter 7 L707

(http://iopscience.iop.org/0953-8984/7/50/002)

J. Phys.: Condens. Matter 7 (1995) L707–L711. Printed in the UK

LETTER TO THE EDITOR

Capillarity phenomenon in a gravity-free zone

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Received 5 September 1995, in final form 18 October 1995

Abstract. The observations related to capillarity made during space flights are explained theoretically. This is achieved by using (i) Young's expression for the contact angle to predict the changing shape of the meniscus and (ii) the Poiseuille–Peiris–Tennakone formula to calculate the speed of the rising column.

In a recent study essay Sally Ride [1] has vividly described many interesting extraterrestrial observations involving surface tension which have been noted in a gravity-free zone realised during space flights. In particular, (i) liquids coalesce into spheres, (ii) a drip would never fall out of a leaky faucet, (iii) near-normal dining with rehydrated food is possible in space, (iv) liquids crawl up a drinking straw and collect in a large drop—a sphere—at the opening of the straw, etc. Upon going through this essay it occurred to us that items (i)–(iii) are essentially self-evident in the absence of gravity. Also, with regard to item (iv) it is expected that for $g \to 0$ the conventional equilibrium height $h \to \infty$, i.e., the liquid will go on rising up to the top of the straw; however, no theoretical explanation has been offered in the literature as far as the formation of a sphere at the opening of the straw is concerned. Obviously, in view of the importance of space flights a theoretical description of item (iv), i.e., capillarity in a gravity-free zone, is worth undertaking. The present letter accomplishes this task.

The formulation of the problem at hand may be conveniently made in terms of the interplay among the adhesive force A and the tensions T_{LV}, T_{SV} and T_{SL} of the liquid–vapour, solid–vapour, and solid–liquid interfaces [2, 3] respectively. From a logical viewpoint the full adhesive force, being a contact force, has a normal component viz. A and also a parallel component which is hidden in the symbol T_{SV}. For the sake of ready reference, let us recapitulate the conditions for these forces to be in equilibrium. Figure 1(a) shows these forces acting on a small element of the liquid situated at the junction of the interfaces and having unit length perpendicular to the plane of the diagram. Resolving these forces parallel and perpendicular to the solid surface, one finds that the said element will be in equilibrium if

$$A = T_{LV} \sin\theta \tag{1a}$$

$$T_{LV} \cos\theta = T_{SV} - T_{SL} \tag{1b}$$

where θ is the equilibrium contact angle. Since (1), known as Young's equation [3], will be of crucial importance in our formalism some pertinent comments on the same are in order. (1a) yields the value of the adhesive force while (1b) may be regarded as defining the equilibrium contact angle in terms of T_{SL}, T_{SV} and T_{LV}. Typical values of the interfacial tensions are reproduced in table 1 to give an idea of their numerical orders of magnitude.

0953-8984/95/500707+05$19.50

Clearly, as long as the interfaces have a nonzero area the contact angle is acute/obtuse for $T_{SV} > / < T_{SL}$. However, due to geometrical considerations an exceptional case may arise when the area of the solid–vapour interface becomes zero. Then, although the value of T_{SV} (which is the free energy per unit area) remains finite the role of the solid–vapour interface disappears from (1*b*) implying that an initially acute θ becomes obtuse. Finally, although (1*b*) was derived for the equilibrium case it also provides a useful guideline to specify the acuteness or obtuseness of the nonequilibrium contact angle for a moving liquid column which will be encountered below.

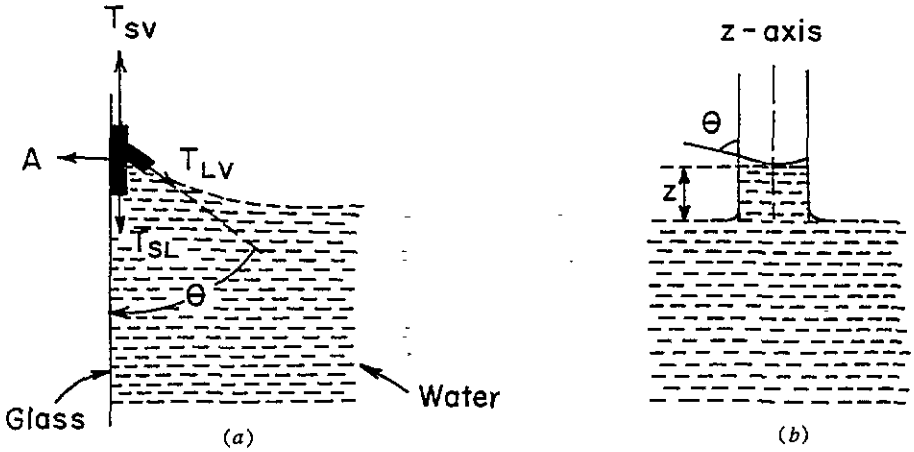

Figure 1. (*a*) A diagram showing the forces *A*, T_{LV}, T_{SV} and T_{SL} acting on a small element of the fluid situated at the junction of the interfaces. (*b*) A diagram showing a capillary tube dipped vertically into a liquid with its lower end just touching the free surface. The contact angle θ and the time dependent rise z are marked.

Table 1. Parameters in the Young equations (in newtons per metre) at room temperature [3]. Here the solid is assumed to be mica and V represents the vapour of the liquid mentioned.

Liquid	T_{SV}	T_{SL}	T_{LV}
Water	0.183	0.107	0.073
Hexane	0.271	0.255	0.018

Suppose a long capillary tube of length L and inner radius r is dipped (see figure 1(b)) vertically into a liquid of surface tension $T \equiv T_{LV}$, contact angle θ ($<\pi/2$), density ρ and viscosity η such that the lower tip of the tube just touches the free surface of the liquid. Let z denote the height of the liquid column above the free surface measured at time t, and let $\dot{z} \equiv \mathrm{d}z/\mathrm{d}t$ denote its speed. A solution to the hydrodynamical problem of capillary motion under the simultaneous influence of surface tension, gravity and viscosity was attempted by the present authors [4] in 1987 using the Newton law for variable-mass systems coupled with an *ad hoc* assumption about the velocity gradient at the wall of the tube. However, since the assumption of the no-slip condition at the wall is more appropriate, one may prefer to employ the Poiseuille formula for the velocity as was done by Peiris and Tennakone [5]. Putting $g = 0$ their formula reads

$$\dot{z} = (rT\cos\theta)/4\eta z. \tag{2}$$

Numerically, for the case of water the relevant typical parameters are as follows:

$$\begin{aligned} &\rho = 10^3 \text{ kg m}^{-3} \\ &r = 10^{-3} \text{ m} \qquad \eta = 10^{-3} \text{ N s m}^{-2} \\ &L = 10^{-1} \text{ m} \qquad T = 0.07 \text{ N m}^{-1}. \end{aligned} \tag{3a}$$

The output Poiseuille–Peiris–Tennakone speeds $\dot{z}_{top}$ at the top of the tube become

$$\dot{z}_{top} = 0.175 \text{ m s}^{-1} \qquad \text{for } \theta = 0 \tag{3b}$$

$$\dot{z}_{top} = 0.166 \text{ m s}^{-1} \qquad \text{for } \theta = 18°. \tag{3c}$$

These two cases correspond to pure water–clean glass and tap water–ordinary glass systems [6], respectively.

We now proceed to an explanation of Sally Ride's observation. As soon as the tube is dipped into water the liquid will start rising rapidly upwards because of the force which the wall exerts on the liquid. The meniscus during this process will remain concave upwards and its speed will be given by (2). Since there is no hindrance from gravity, the column will go on rising until the meniscus approaches the upper tip of the tube. Geometrical considerations now become important. As the liquid fills the complete inner volume of the tube the vertical solid–vapour film disappears so that the contact angle tends to change from an acute to obtuse value as mentioned above. Now, due to inertia the column overshoots the tip and also starts spreading over it because of the reappearance of the solid–vapour interfacial tension now acting in the horizontal direction. This spreading is expected to be rapid and nonstreamline so the kinetic energy gained by the liquid head will be partly dissipated in overcoming viscosity. Of course, during the horizontal spreading the directions of various interfacial tensions have changed compared to their directions during the rising stage mentioned above. Two cases need to be distinguished at this juncture depending upon the thickness of the wall.

If the tube has a rather thin wall the spreading liquid overshoots even the outer periphery P (see figure 2(a)) of the tube and the subsequent events happen in a manner analogous to the discussion after (1). That is, as the solid–vapour interface effectively disappears the contact angle changes from acute to obtuse momentarily as the liquid turns around P. This is followed by a small spreading on the outer surface of the straw resulting in a final equilibrium configuration of the liquid head subject to the following requirements: (i) in order to have minimum surface area the liquid takes a predominantly spherical shape truncated by the outer periphery of the tube, and (ii) exactly at the junction of the three interfaces there will be a slight distortion of the spherical geometry of the drop in order to allow for the retrieval of the standard contact angle given by (1). This is consistent with the observations reported by Sally Ride [1] during space flights.

One may wonder what is the size of the blob so formed. To answer this, we recall that the liquid column had the kinetic energy $\frac{1}{2}\pi r^2 L\rho\dot{z}_{top}^2$ just before crossing the top of the tube. This kinetic energy is mostly converted into the surface energy E_S (say) of the blob and a smaller part E_V (say) is dissipated due to viscosity. First, let us calculate E_S by remembering that, in figure 2(a), the spherical blob of radius R is truncated by the outer periphery of the capillary tube with outer radius r_{out}. Multiplying the surface area of this truncated sphere by T and equating it to the kinetic energy of the liquid column, we obtain

$$\pi\left[4R^2 - r_{out}^2 - \mathrm{O}\left(r_{out}^4/R^2\right)\right]T = \tfrac{1}{2}\pi r^2 L\rho\dot{z}_{top}^2. \tag{4}$$

Inclusion of the comparatively smaller contribution from E_V will alter (4) somewhat in a

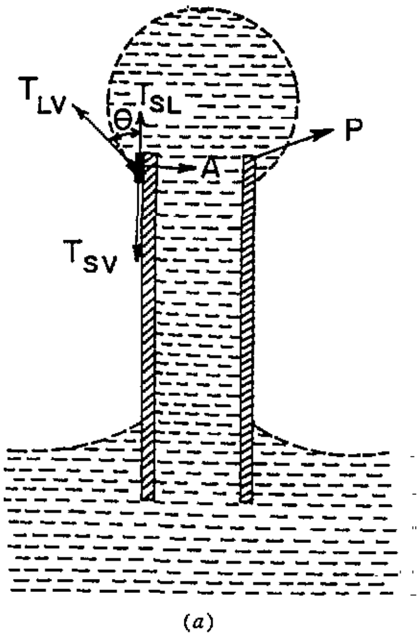

Figure 2. The rise of water in a capillary tube when gravity is absent. (*a*) the wall of the tube is rather thin and the liquid forms a sphere around the tip. The outer periphery is marked by the point *P*. (*b*) The wall of the tube is rather thick and the liquid comes to rest over the rim as shown.

manner mentioned after the numerical estimates given below. Solving (4) we obtain

$$R = \frac{r}{2}\left[\frac{r^2 \rho T \cos^2\theta}{32\eta^2 L} + 1\right]^{1/2} \tag{5}$$

where r_{out} has been set equal to r for a thin tube. Next, let us derive an expression for E_V which is approximately equal to the product of the viscous drag force F_V at the top of the tube and an additional vertical displacement ΔZ of the liquid head before coming to rest. Clearly, $F_V = 8\pi\eta L\dot{z}_{top}$ by Poiseuille's theory while ΔZ is of the order of R. Multiplying them we arrive at

$$E_V \sim 8\pi\eta L\dot{z}_{top}R. \tag{6}$$

Let us now turn to numerical estimates. Using the data of (3*a*–*c*) we obtain for $\theta = 0$, i.e., the pure water—clean glass system

$$\begin{aligned} R &= 2.39 \times 10^{-3}\ \mathrm{m} \qquad & E_V &\sim 1.05 \times 10^{-6}\ \mathrm{J} \\ E_S &= 4.81 \times 10^{-6}\ \mathrm{J} \qquad & E_V/E_S &\sim 0.22. \end{aligned} \tag{7a}$$

Next, for $\theta = 18°$, i.e., the tap water–ordinary glass system

$$\begin{aligned} R &= 2.28 \times 10^{-3}\ \mathrm{m} \qquad & E_V &\sim 9.51 \times 10^{-7}\ \mathrm{J} \\ E_S &= 4.33 \times 10^{-6}\ \mathrm{J} \qquad & E_V/E_S &\sim 0.22. \end{aligned} \tag{7b}$$

From (7*a*) and (7*b*) two important observations are made. Firstly, the ratio E_V/E_S is about 0.2. Thus, the kinetic energy will be converted into the sum $E_S + E_V \sim 1.2E_S$ and, in turn, this quantity will now appear on the left-hand side of (4). Since the radius of the blob is proportional to the square root of its surface area, the new magnitude of R will be $(1.2)^{1/2} \sim 0.9$ times the values reported in (7a) and (7b). Secondly, the magnitude of R turns out to be more than twice the value of r which is expected for a thin tube. It may be added that the possibility of the liquid crawling back down the outside of the tube is remote because the sphere is the most stable configuration which has been formed satisfying energy conservation (cf. (4)). Also, one may wonder whether due to the reverse curvature of, and hence due to the excess pressure $(2T/R)$ within, the drop the liquid will be pushed back down the tube! This, however, cannot happen because the drinks used by astronauts come in vacuum packed, plastic cartons with thin plastic tops [1] and the above mentioned excess pressure within the drop will not be able to overcome the tension of plastics. Therefore, the blob observed by Sally Ride is a stable configuration rather than a metastable one ignoring, of course, evaporation effects.

If the outer radius of the tube is large compared to the radius parameter R defined in (5) the tube may be called a *thick one*. In such a case, the spreading water does not reach the outer periphery of the tube and it comes to rest with the liquid protruding out in the form of a spherical shell balancing over the rim as shown in figure 2(b). The equilibrium conditions are retrieved in the form (1) with the contact angle having become acute once again and the kinetic energy of the liquid head is mostly consumed into the surface energy of the protrusion.

Thanks are due to P Khastgir for stimulating discussions and to the UGC for financial support. The authors are grateful to the referees for their constructive suggestions.

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